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The Fixed-Point Algebra of Tensor-Product Actions of Finite Abelian Groups on UHF-Algebras

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Let G be a finite Abelian group acting by tensor-product automorphisms on a UHF-C^*-algebra $\mathscr{D}$. Extending a result of A. Kishimoto it is shown that the number of extremal traces on the fixed-point algebra $\mathscr{D}^G$ equals the cardinality of the subgroup K of automorphisms in G which are weakly inner in the trace representation of $\mathscr{D}$.

1. INTRODUCTION

In [1] Kishimoto investigated the properties of tensor-product actions of the groups $G = \mathbb{Z}_n$ on UHF-algebras $\mathscr{D}$. Among other things, he showed that the number of extremal traces on the fixed-point algebra $\mathscr{D}^G$ equals the cardinality of a subgroup K of G.

The method employed here leans on that of Kishimoto in [1], especially his Lemma 3.9. But a certain limit result (Lemma 2.2) allows us for finite Abelian G to identify K as the subgroup of weakly inner automophisms in the trace-representation. Further the extremal traces are precisely τ_i, $1 \leqslant i \leqslant |K|$, given in the trace representation by $\tau_i(x) = \tau(xE_i)$, where $|\ |$ denotes cardinality, τ is the canonical trace on the strong closure of $\mathscr{D}$ and the E_i, $1 \leqslant i \leqslant |K|$, are the minimal projections in the center of the strong closure of $\mathscr{D}^G$ (Theorem 4.2 and Corollary 4.3).

2. NOTATION AND PRELIMINARIES

Let G be a finite Abelian group and let K_n be matrix factors. Consider unitary representations $\pi_n: G \to K_n$ and define the action α of G on $\mathscr{D} = \otimes_{n=1}^{\infty} K_n$ by $\alpha_g = \otimes_{n=1}^{\infty} \mathrm{Ad}(\pi_n(g))$.

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We take the notation $W_g^{n,m} = \otimes_{i=n}^m \pi_i(g)$, $n \leqslant m$. Decomposing unitary representations, put $W_g^{n,m} = \sum_{\lambda \in \hat{G}} \lambda(g) E_\lambda^{n,m}$ with $E_\lambda^{n,m}$ projections in $\otimes_{i=n}^m K_i$ and $\hat{G}$ the character group of G. Note that $E_\lambda^{n,m} = \sum_{\eta \in \hat{G}} E_{\lambda\eta^{-1}}^{n,s} E_\eta^{s+1,m}$, $n \leqslant s < m$.

The criterion [6, Lemma 3.5] shows, that α_g is inner, if and only if $\mathrm{Ad}(\pi_n(g))$ differs from the identity for only finitely many n. We assume throughout that $H = \{g \in G \mid \alpha_g \text{ is inner}\} = \{e\}$ and may then further assume all $E_\lambda^{m,n}$ to be nonzero—changing notation if necessary, $K_1' = \otimes_{i=1}^{n_1} K_i$, $K_2' = \otimes_{i=n_1+1}^{n_2} K_i$ a.s.o.

By [5, Lemma 5.3] $\mathscr{D}^G = \{x \in \mathscr{D} \mid \alpha_g(x) = x, \ \forall g \in G\}$ is equal to $(\bigcup_{n=1}^\infty \{x \in \otimes_{i=1}^n K_i \mid \alpha_g(x) = x, \ \forall g \in G\})^-$ ($^-$ denoting norm closure). Putting $A_\lambda^n = E_\lambda^{1,n} (\otimes_{i=1}^n K_i) E_\lambda^{1,n}$, a matrix factor, this yields $\mathscr{D}^G = (\bigcup_{n=1}^\infty [\sum_{\lambda \in \hat{G}} \oplus A_\lambda^n])^-$, an AF-algebra.

Define $|\ |$ to be the rank on matrix factors, i.e., $|M_n(\mathbb{C})| = n$, and let τ be the canonical trace on $\mathscr{D}$.

We refer to [5, p. 198–199; 8, Lemma 2.2] for description of how one finite-dimensional C^*-algebra embeds into another, and for explanation of the term "multiplicity of a partial embedding."

LEMMA 2.1. *The partial embedding* $A_\lambda^n \to A_\mu^{n+1}$ *has multiplicity* $|K_{n+1}|\, \tau(E_{\mu\lambda^{-1}}^{n+1,n+1})$.

Proof. Let $x \in A_\lambda^n$. Then $x E_\mu^{1,n+1} = x \sum_{\eta \in \hat{G}} E_\eta^{1,n} E_{\mu\eta^{-1}}^{n+1,n+1} = x E_{\mu\lambda^{-1}}^{n+1,n+1}$. The embedding has form $x \to x E_{\mu\lambda^{-1}}^{n+1,n+1}$ and multiplicity

$$\left(\tau(E_\lambda^{1,n} E_{\mu\lambda^{-1}}^{n+1,n+1}) \left| \bigotimes_{i=1}^{n+1} K_i \right| \right) \left(\tau(E_\lambda^{1,n}) \left| \bigotimes_{i=1}^{n} \right| \right)^{-1} = \tau(E_{\mu\lambda^{-1}}^{n+1,n+1})\, |K_{n+1}|. \quad \blacksquare$$

In the representation $(\pi_\tau, \mathscr{H}_\tau, \xi_\tau)$ associated with the canonical trace τ on $\mathscr{D}$ one gets a unitary representation $u: \varphi \in \mathrm{Aut}(\mathscr{D}) \to u_\varphi \in B(\mathscr{H}_\tau)$ such that $\varphi = \mathrm{Ad}(u_\varphi)$, through the expression $u_\varphi(\pi_\tau(x)\xi_\tau) = \pi_\tau(\varphi(x))\xi_\tau$.

Consequently each $\varphi \in \mathrm{Aut}(\mathscr{D})$ is uniquely extendable to $\tilde{\varphi} \in \mathrm{Aut}(R)$, $R = \pi_\tau(\mathscr{D})''$ being the hyperfinite II_1-factor. We identify $\mathscr{D}$ and $\pi_\tau(\mathscr{D})$. We shall need the following, which is well known.

LEMMA 2.2. *Let* $u_n \in K_n$ *be unitaries and define* $\theta \in \mathrm{Aut}(\mathscr{D})$ *by* $\theta = \otimes_{n=1}^\infty \mathrm{Ad}(u_n)$. *Then* $\tilde{\theta}$ *is inner if and only if there is an* m *such that* $\prod_{n=m}^\infty |\tau(u_n)| \in\,]0, 1]$.

If in this case $\prod_{n=m}^\infty \tau(u_n) \in \mathbb{C} \setminus \{0\}$, *we even get* $\theta = \mathrm{Ad}(\mathrm{st}^* - \otimes_{n=1}^\infty u_n)$ st^* *indicating convergence in the strong-*-topology.*

Sketch of Proof. If $\prod_{n=m}^{\infty} \tau(u_n) \in \mathbb{C}\setminus\{0\}$ one may directly construct $W = st - \otimes_{n=1}^{\infty} u_n$ using

$$\left\| \left(\bigotimes_{l=1}^{n+s} u_l - \bigotimes_{l=1}^{n} u_l \right) \left(\bigotimes_{i=1}^{p} a_i \right) \xi_\tau \right\|^2$$

$$= 2 \left(1 - \operatorname{Re} \left(\prod_{l=n+1}^{n+s} \tau(u_l) \right) \right) \left(\prod_{i=1}^{p} \tau(a_i^* a_i) \right)$$

for $p < n$, $s \geqslant 0$, and $a_i \in \mathscr{D}$. Similarly $W^* = st - \otimes_{n=1}^{\infty} u_n^*$. If on the other hand $\tilde{\theta} = \mathrm{Ad}(W)$, $W \in R$, choose $a = a_1 \otimes \cdots \otimes a_n$, $a_i \in K_i$, so that $\tau(aW) \neq 0$. As $(u_1 \cdots u_l)^{-1} W \in (\otimes_{i=1}^{l} K_i)'$, we have for $l > n$

$$0 < |\tau(aW)| = \left| \left(\prod_{i=1}^{n} \tau(a_i u_i) \right) \left(\prod_{i=n+1}^{l} \tau(u_i) \right) \tau((u_1 \cdots u_l)^{-1} W) \right|$$

$$\leqslant \left(\prod_{i=1}^{n} |\tau(a_i u_i)| \right) \left(\prod_{i=n+1}^{l} |\tau(u_i)| \right)$$

and then $\prod_{i=n+1}^{\infty} |\tau(u_i)| > 0$. ∎

3. A Limit Result

Let $K = \{ g \in G \mid \tilde{\alpha}_g \text{ is inner} \}$. After "compressing" the system $\{K_n\}_{n=1}^{\infty}$, renaming $K_1' = \otimes_{n=1}^{i_1} K_n$, $K_2' = \otimes_{n=i_1+1}^{i_2} K_n$, and so on, we may by means of Lemma 2.2 require that $\tilde{W}_k := \text{st}^*\text{-lim}_{n \to +\infty} W_k^{1,n}$ is a unitary representation of K in $R^G = \{x \in R \mid \tilde{\alpha}_g(x) = x,\ \forall g \in G\}$, such that $\tilde{\alpha}_k = \mathrm{Ad}(\tilde{W}_k)$, $k \in K$. Likewise $\tilde{W}_k^s := \text{st}^*\text{-lim}_{s \leqslant n \to +\infty} W_k^{s,n}$ implements $\alpha_k|_{\otimes_{i=s}^{\infty} K_i}$.

Identifying $\hat{K}$ with a choice of one representative of each coset of $K^\perp = \{\gamma \in \hat{G} \mid \gamma(K) = 1\}$ in $\hat{G}$ and putting $E_{[\lambda]}^{m,n} = \sum_{\eta \in K^\perp} E_{\lambda\eta}^{m,n}$, one has $W_k^{m,n} = \sum_{\lambda \in \hat{K}} \lambda(k) E_{[\lambda]}^{m,n}$ and $E_{[\lambda]}^{m,n} = \sum_{k \in K} \overline{\lambda(k)}\, W_k^{m,n}$. Then the projection $E_{[\lambda]}^{m,\infty} := \text{st}^*\text{-lim}_{m \leqslant n \to +\infty} E_{[\lambda]}^{m,n}$ is well defined and $\tilde{W}_k^m = \sum_{\lambda \in \hat{K}} \lambda(k) E_{[\lambda]}^{m,\infty}$. The identity $E_\lambda^{m,n} = \sum_{\eta \in \hat{G}} E_\eta^{m,s} E_{\lambda\eta^{-1}}^{s+1,n}$, $m \leqslant s < n$, implies that $E_{[\lambda]}^{m,\infty} = \sum_{\eta \in \hat{K}} E_{[\eta]}^{n,s} E_{[\lambda\eta^{-1}]}^{s+1,\infty}$, and that $E_{[\lambda]}^{m,\infty} \neq 0$, $\forall m$, $\forall \lambda$, as $\inf_{\lambda \in \hat{G}} \tau(E_\lambda^{m,n}) > 0$.

Let $|\ |$ denote cardinality, and let $1_{K^\perp}$ be the characteristic function of $K^\perp$.

Proposition 3.2. (a) $\tau(E_\lambda^{m,n}) \to_{n \to +\infty} |K|/|G|\, \tau(E_{[\lambda]}^{m,\infty})$, $\forall m \in \mathbb{N}$, $\forall \lambda \in \hat{G}$.

(b) $\tau(E_{[\lambda]}^{m,\infty}) \to_{m \to +\infty} 1_{K^\perp}(\lambda)$, $\forall \lambda \in \hat{G}$.

Proof. Part (b) is a restatement of $\tau(\tilde{W}_k^m) \to_{m \to +\infty} 1$, $\forall k \in K$. To prove (a), we may take $m = 1$. For brevity put

$$E_{[\lambda]}^{1,\infty} = E_{[\lambda]}, \qquad E_{[\lambda]}^{1,n} = E_{[\lambda]}^n, \qquad E_\lambda^{1,n} = E_\lambda^n.$$

We seek to employ the fact, that G/K has outer action on $E_{[\lambda]}RE_{[\lambda]}$, (cf. [7]). Let $x \in \otimes_{i=1}^{n} K_i$ and $g \in G/K$.

$$\begin{aligned}
\tilde{\alpha}_g(E_{[\lambda]}xE_{[\lambda]}) &= \tilde{\alpha}_g\left(\sum_{\eta\in\hat{K}} E^n_{[\eta]}xE^n_{[\eta]}E^{n+1,\infty}_{[\lambda\eta^{-1}]}\right)\\
&= \sum_{\substack{\eta\in\hat{K}\\ \mu,\xi\in\hat{G}}} \mu(g)\,\xi(g^{-1})\,E^n_{[\eta]}E^n_{\mu}xE^n_{\xi}E^n_{[\eta]}E^{n+1,\infty}_{[\lambda\eta^{-1}]}\\
&= \sum_{\substack{\eta\in\hat{K}\\ \mu,\xi\in K^{\perp}}} \mu(g)\,\xi(g^{-1})\,E^n_{\eta\mu}xE^n_{\eta\xi}E^{n+1,\infty}_{[\lambda\eta^{-1}]}\\
&= \sum_{\substack{\eta_1,\eta_2,\eta_3,\eta_4\in\hat{K}\\ \mu,\xi\in K^{\perp}}} \mu(g)\,\xi(g^{-1})\,E^{n+1,\infty}_{[\lambda\eta_2^{-1}]}E^n_{\eta_2\mu}E^n_{[\eta_3]}E^{n+1,\infty}_{[\lambda\eta_3^{-1}]}\\
&\qquad xE^n_{[\eta_4]}E^{n+1,\infty}_{[\lambda\eta_4^{-1}]}E^n_{\eta_1\xi}E^{n+1,\infty}_{[\lambda\eta_1^{-1}]}\\
&= \mathrm{Ad}\left(\sum_{\mu\in K^{\perp}}\mu(g)\left(\sum_{\eta\in\hat{K}} E^n_{\eta\mu}E^{n+1,\infty}_{[\lambda\eta^{-1}]}\right)\right)(E_{[\lambda]}xE_{[\lambda]}).
\end{aligned}$$

Call the $E_{[\lambda]}RE_{[\lambda]}$-unitary in brackets: $V^n_{\lambda}(g)$. Outerness of $\tilde{\alpha}_g$, $g \neq e$, on $E_{[\lambda]}RE_{[\lambda]}$ implies $\tau(V^n_{\lambda}(g)) \to_{n\to+\infty} 1_e(g)\,\tau(E_{[\lambda]})$. To see this let $g \neq e$, put $V^n_{\lambda}(g) = V^n$ and assume ad absurdum without loss of generality $|\tau(V^n)| \geqslant \varepsilon > 0$ and $V^n \to^{\text{weakly}}_{n\to+\infty} V$. The calculation for

$$\begin{aligned}
y \in E_{[\lambda]}\left(\bigotimes_{i=1}^{n_0} K_i\right)E_{[\lambda]} &: \tilde{\alpha}_g(y)VV^*\\
&= \underset{n\to+\infty}{w\text{-lim}}\ \tilde{\alpha}_g(y)\,V^nV^* = \underset{n\to+\infty}{w\text{-lim}}\ V^nyV^* = \cdots = VV^*\tilde{\alpha}_g(y),
\end{aligned}$$

shows $VV^* \in (\bigcup_{n_0=1}^{\infty} E_{[\lambda]}(\otimes_{i=1}^{n_0} K_i)\,E_{[\lambda]})' \cap E_{[\lambda]}RE_{[\lambda]} = \mathbb{C}E_{[\lambda]}$. Similarly $V^*V \in \mathbb{C}E_{[\lambda]}$. Consequently $V = \lambda u$ for some unitary u and $\lambda > 0$. When $x \in E_{[\lambda]}(\otimes_{i=1}^{n_0} K_i)\,E_{[\lambda]}$ the equality $\tilde{\alpha}_g(x)V^n = V^nx$, which holds for $n \geqslant n_0$, yields $\tilde{\alpha}_g(x) = \mathrm{Ad}(u)(x)$. u is seen to implement $\tilde{\alpha}_g$, and we have a contradiction.

Now $\tau(V^n_{\lambda}(g)) \to 1_e(g)\,\tau(E_{[\lambda]})$ implies by Fourier transforming

$$\sum_{\eta\in\hat{K}} \tau(E^n_{\eta\mu})\,\tau(E^{n+1,\infty}_{[\lambda\eta^{-1}]}) \xrightarrow[n\to+\infty]{} |G/K|^{-1}\tau(E_{[\lambda]}), \qquad \forall\mu\in K^{\perp}, \forall\lambda\in\hat{G}.$$

However, employing (b) we have

$$\sum_{\eta\in\hat{K}} \tau(E^n_{\eta\mu})\,\tau(E^{n+1,\infty}_{[\lambda\eta^{-1}]}) - \tau(E^n_{\lambda'\mu}) \xrightarrow[n\to+\infty]{} 0, \qquad \forall\mu\in K^{\perp}, \forall\lambda\in\hat{G},$$

where λ' is an element of $\hat{G}$ equal to λ in $\hat{G}/K^{\perp}$. This proves (a). ∎

4. Extremal Traces on $\mathscr{D}^G$

Let τ' be a (unnormalised) trace on $\mathscr{D}^G$. Then chosing a minimal projection F_λ^n in A_λ^n (cf. Sect. 2), τ' is by continuity completely determined through the nonnegative numbers $\tau'(F_\lambda^n)$, $n \in \mathbb{N}$, $\lambda \in \hat{G}$. By Lemma 2.1 the embedding $A_\lambda^{n-1} \to A_\mu^n$ has multiplicity $|K_n|\, \tau(E_{\mu\lambda^{-1}}^{n,n})$, so letting μ vary $\tau'(F_\lambda^{n-1}) = \sum_{\mu \in \hat{G}} |K_n|\, \tau(E_{\mu\lambda^{-1}}^{n,n})\, \tau'(F_\mu^n)$. That is, letting $*$ be convolution $f * g(\lambda) = \sum_{\mu \in \hat{G}} f(\mu)\, g(\mu^{-1}\lambda)$, and $\check{}$ the flip $\check{f}(\lambda) = f(\lambda^{-1})$, we get $\tau'(F_\cdot^{n-1}) = |K_n|\, \tau(E_\cdot^{n,n})\check{} * \tau(F_\cdot^n)$.

Lemma 4.1. *There is a one to one correspondence between traces τ' on $\mathscr{D}^G$, and sequences $\{f_n\}_{n=1}^\infty$ of nonnegative functions on $\hat{G}$ fulfilling*

$$f_n * \tau(E_\cdot^{n,n})\check{} = f_{n-1} \tag{$*$}$$

given by

$$f_n(\lambda) = \left| \bigotimes_{i=1}^n K_i \right| \tau'(F_\lambda^n). \tag{$**$}$$

Proof. If τ' is given and $\{f_n\}_{n=1}^\infty$ are defined by $(**)$, Eq. $(*)$ is a consequence of the remarks above. If conversely $\{f_n\}_{n=1}^\infty$ are given, $(*)$ is the condition that $(**)$ define a trace on $\mathscr{D}^G$. ∎

We may now prove the theorem.

Theorem 4.2. *The normalised traces on $\mathscr{D}^G$ form a simplex with exactly $|K|$ extremal traces, these being given by $\tau_\mu(x) = \tau(E_{[\mu]}^{1,\infty})^{-1}\tau(xE_{[\mu]}^{1,\infty})$, $\mu \in \hat{K}$.*

Proof. Let $a_n(\lambda) = |K|/|G|\, \tau(E_{[\lambda]}^{n,\infty})$ and note that $a_{n-1} = a_n * \tau(E_\cdot^{n-1,n-1})\check{}$, $n \geqslant 2$. We define a mapping L from $L^2(\hat{G})_+$ into the trace-space of $\mathscr{D}^G$, letting $L(f)$ be the trace associated the sequence $\{f * (a_{n+1})\check{}\}_{n=1}^\infty$ in the sense of Lemma 4.1.

This is a surjective map. Let τ' corresponding to $\{f_n\}_{n=1}^\infty$ be given. One sees $\|f_n\|_\infty \leqslant \|f_n\|_1 = \|f_1\|_1$, and may choose a sequence $\{k_p\}_{p=1}^\infty$, so that $f'(\lambda) := \lim_{p \to +\infty} f_{k_p}(\lambda)$ exists. Clearly $f' * (a_{n+1})\check{} = f_n$, and $\tau' = L(f')$.

Considering only real-valued functions on $\hat{G}$, extend L to $L^2(\hat{G})$. Let $\hat{}$ denote the Fourier transform $L^2(\hat{G}) \to L^2(G)$. We have $\tau(E_\cdot^{n,m})\hat{} = \tau(W_\cdot^{n,m})\check{}$. Therefore by Proposition 3.2(a), the function $(a_n\check{})\hat{}$ is given by $g \to \lim_{m \to +\infty} \tau(W_g^{n,m})$, and $|(a_n\check{})\hat{}| \nearrow_n 1_K$ by Lemma 2.2. Hence $L(f) = 0$ if and only if $\hat{f}|_K = 0$. The orthogonal complement to the set of $\hat{f} \in L^2(G)$ such that $\hat{f}|_K = 0$, is the set of $\hat{f} \in L^2(G)$ such that $\hat{f}|_{G \setminus K} = 0$. Letting $L^2(\hat{G}/K^\perp)$ denote the subspace of functions in $L^2(\hat{G})$ constant on the cosets of $K^\perp$, one sees $\hat{f}|_{G\backslash K}=0 \Leftrightarrow f \in L^2(\hat{G}/K^\perp)$, so $\ker L$ is the orthogonal complement of $L^2(\hat{G}/K^\perp)$ in $L^2(G)$. The projection $L^2(G) \to L^2(\hat{G}/K^\perp)$ is positive, thus L restricts to an order isomorphism between $L^2(\hat{G}/K^\perp)_+$ and the traces on $\mathscr{D}^G$. The extremal traces in the convex set of traces τ' with $\|\tau\| = \tau'(I) \in [0, 1]$, are $\tau_\mu = L(1_{\mu K^\perp})(I)^{-1} L(1_{\mu K^\perp})$, $\mu \in \hat{K}$.

Verification of $(1_{\mu K^\perp} * \check{a}_{n+1})(\lambda) = \tau(E^{n+1,\infty}_{[\mu\lambda^{-1}]})$ yields

$$L(1_{\mu K^\perp})(F^n_\lambda) = \left|\bigotimes_{i=1}^{n} K_i\right|^{-1} \tau(E^{n+1,\infty}_{[\mu\lambda^{-1}]}) = \tau(F^n_\lambda E^{n+1,\infty}_{[\mu\lambda^{-1}]}) = \tau(F^n_\lambda E^{1,\infty}_{[\mu]})$$

and $\tau_\mu(x) = \tau(E^{1,\infty}_{[\mu]})^{-1} \tau(x E^{1,\infty}_{[\mu]})$ as claimed. ∎

COROLLARY 4.3. *The center of R^G is $|K|$ dimensional.*

Proof. Clearly $\oplus_{\eta \in \hat{K}} \mathbb{C} E^{1,\infty}_{[\eta]} \subseteq Z(R^G)$. But if F is a central projection $\tau(\cdot\, F) = \sum_\eta \alpha_\eta \tau(\cdot\, E^{1,\infty}_{[\eta]})$ implies $F = \sum_\eta \alpha_\eta E^{1,\infty}_{[\eta]}$. ∎

Remark A. Corollary 4.3 may also be shown using outerness of G/K on all $E^{1,\infty}_{[\eta]} R E^{1,\infty}_{[\eta]}$ as in [3, 2.1.1–2.1.3].

Remark B. Relaxing the assumption $H = \{g \in G \mid \alpha_g \text{ is inner}\} = \{e\}$, to just α being injective, we may assume $E^{1,m}_\lambda \neq 0$ $\forall m$, $\forall \lambda$, and $E^{n,m}_\lambda \neq 0$ exactly when $\lambda \in H_0$, for $2 \leqslant n \leqslant m$, with H_0 a subgroup of $\hat{G}$ (cf. [4, Sect. 3]). Clearly $H_0^\perp = H$. (See, e.g., 6, Lemma 3.5]). Put $P_\lambda = \sum_{\mu \in H_0} E^{1,1}_{\lambda\mu}$. The group G/H has outer action on the UHF-algebra $P_\lambda \mathscr{D} P_\lambda = (P_\lambda K_1 P_\lambda) \otimes (\otimes_{n=2}^\infty K_n)$ and applying Theorem 4.2 and [5, Lemma 3.2] in combination with (the analogous statement of) Lemma 2.1, we find that each $(P_\lambda \mathscr{D} P_\lambda)^{G/H} = P_\lambda \mathscr{D}^G$ is a simple AF-algebra with $|K|/|H|$ extremal traces, and that $Z(\mathscr{D}^G) = \oplus_{\lambda \in \hat{H}} \mathbb{C} P_\lambda$.

Remark C. Neither method nor result of this paper extends to the case where G is, say, Abelian and compact. Take G to be the circle group and α the gauge action on $\mathscr{D} = \otimes_{n=1}^\infty M_2^{(n)}$. Then K is the singleton $\{e\}$. Nevertheless the extremal traces on $\mathscr{D}^G$ form a one-parameter family (cf. [2, p. 148]).

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